TYPES OF MATHEMATICIANS

Coralie Marshall
University of Otago

Abstract

This paper reports on research being conducted in the area of the identification of characteristics of mathematical ability. Research is being carried out through case studies of ten students with ability in mathematics, with input from their parents, siblings, teachers and peers, as well as from the students themselves. The age of the students in the sample, has allowed observations about the them to cover the period spanning from infancy to tertiary levels of education, and the gap between the events and their reporting has been relatively short. Differences in approach between the students have been identified. These are compared with differences noted in the research of Krutetskii and Osborn, and comment is made on some of the implications that such differences in approach and aptitude have for mathematics education.

Introduction

The study reported in this paper arose out of a survey conducted by the Department of Mathematics and Statistics of the University of Otago, of those students who had attended an New Zealand Maths Olympiad Committee camp between 1987 and 1991, and of their parents. Just over 70% of the students and parents contacted, responded to the survey. The information received raised a number of issues and indicated areas for further research. For example, many of the students said that school mathematics was too easy, repetitive and boring, and stories of unhelpful classroom experiences were frequently reported. Parents had often been aware of their child's ability and frustration, but had been unable to access the help that they felt was needed. One parent seemed to sum up the opinion of many of the parents when she commented, "While teachers were convinced of his ability, the system did not seem flexible enough to cater for him." Many of the parents felt that their child had been self-motivated in their interest in mathematics, and some ventured the opinion that it almost seemed inborn. Interesting differences in attitudes to mathematics, and to other subjects, also emerged. (See Curran, Holton, Marshall, Pek, 1992.) Because there was a good deal of common ground in responses, it was decided to continue the research by selecting ten students from within the group, and studying them in greater detail.

Methodology

<u>**Selection of the sample.**</u> The ten students selected had all been identified initially by achieving high marks in either the Westpac or National Bank Maths Competitions, or both. Eight of the ten had been selected for inclusion in an New Zealand International Maths Olympiad team. Five had completed a degree course by the time the study began. All ten students had appeared from the questionnaire responses, to have a variety of interests, an ability in other subjects as well as in mathematics, a willingness to respond to challenge, and a good rapport with peers and family.

The gender ratio in the study was eight males to two females; in the original survey it was eight to one. One student was an only child, two came from families of two, five from families of three, one from a family of four, and one from a family of five. Four students were first-born in their families, three were second-born, and three were both third-born and the youngest in their family.

One student had had most of his education in a remote Area School. One had been educated at both primary and secondary level in single sex private schools. One female and three males had had their secondary schooling in single sex state schools, and one female and three males had attended co-educational state secondary schools. Seven of those eight had attended average sized suburban primary and intermediate schools. The eighth had attended a private primary school, after the first two years of schooling.

Seven of the fathers came within the top three of the five groups of the Elley-Irving Socio-Economic Index. These three groups together represent 42% of the male labour force in New Zealand when it is calculated on the basis of the median educational and income levels for males aged 25-44 years (Elley and Irving, 1985). However, for a number of reasons, not all of these families had had the sort of disposable income, during the student's up-bringing, that those figures would suggest.

Nine of the mothers and seven of the fathers were graduates. Six of these mothers and two of the fathers had degrees in Arts. Three of the parents had degrees in Science. One of these, a mother, had a degree in mathematics. During the selection of the sample, the educational backgrounds of the parents of two of the students were taken into account intentionally, to ensure that all of the sample did not have graduate parents. In one case neither parent had any tertiary education. In the second case, the mother of the family had left school at fifteen and pursued further education only after she had had her family of five children. The father of that family had a technical certificate.

Case Studies. The study was based around case studies and designed to include a study of the family, the home environment, and the educational experiences of each student. A number of researchers, for example, Bloom (1985), Moore and Morrison (1988), and Gross (1993), have used this approach successfully. Gross recognised its ability to present a holistic view of individuals or events through the variety of the evidence it can employ and quoted Foster (1986) in describing the approach as "ideally suited to the investigation and description of events or individuals characterized by their rarity, such as exceptionally or profoundly gifted children" (Gross, 1993, p62).

By using the case study method this study was able to add detail to the data already compiled through the original survey, about a group who had completed their undergraduate education, but were young enough for there to be a limited gap between the events mentioned and their reporting. Because of the number of students and interviews that could be handled when a study of this breadth was decided upon, no attempt was made to claim the study as a statistically balanced experiment.

The ten students named parents, brothers and sisters, friends and/or peers, and teachers whom they thought would be able to add illuminating information. This research is still in progress, and some follow-up interviews and tests have still to be made, but so far ten students, sixteen parents, nine brothers and sisters, seventeen friends (all of whom had been in the same class as the survey student at some stage during schooling), and twenty-six teachers (eleven of whom were primary school teachers, ten secondary school teachers, and five university staff) have been interviewed.

A list of areas of interest had been prepared prior to the interviews, and some specific questions were asked. But usually interviewees were encouraged to talk freely. Interviews usually varied in length from thirty minutes to one hour with friends and teachers, and from one to three hours with the students in the study and with their parents and siblings. Gross (1993) reports that semi-structured interviews of this type have been identified as appropriate in the area of education (Borg and Gall, 1983), and that it is recognised that neither the exact wording nor the order of the questions have to be adhered to in the interviews (Merriam, 1988).

Discussion

The student mathematicians in the study had been selected from what was initially a very large sample base, through the same basic tests and competitions. The degree of difficulty of at least the final test, in the process towards selection for the NZMOC camp, in itself classified them as extremely able mathematicians. All were described by parents and teachers alike as having a high level of concentration, a noticeable self-sufficiency, an ability to be absorbed in activities, often undertaken on their own initiative. Although few were outstanding at team sports, most had been eager to join in physical activities, and often showed particular aptitude in individualistic sports. In other words, they appeared to have shown many of the characteristics frequently listed as those of gifted children (for example, those listed by Kerry, 1983) as well as the two other characteristics, of task commitment and creativity, added by Renzulli (1977) in his "three-ring" concept of giftedness.

Different types of mathematicians. When the study began, it was expected that there would be characteristics which were common to at least most of the students in the study, and that basically different characteristics would be found in those siblings and/or friends who were not mathematicians or who were but had not achieved the same success. But from a very early stage in the interviewing process, differences between the ten students themselves began to emerge as well. By the time the interviews were completed the ten students had largely divided themselves into three groups with some significant and distinguishable characteristics.

The groups were given the names Group S (because they had many of the characteristics that were usually associated with **spatial** abilities), Group R (because they gave brief, **rationalised** answers about what they were doing rather than detailed descriptions), and Group P (simply because their approach to a number of things seemed to be more **pragmatic** than was the case with those in Groups S or R). There were differences within the groups, of course: no two students were exactly alike. One was tempted to see the differences as movement along a horizontal line from one extreme of mathematical preference and ability, to the other. However, there were sufficient differences, in the way both mathematics and other subjects were approached, to follow the inclination to group students even though the possibility of transition between groups clearly existed.

Six students largely fitted into Group S. They had a high level of ability to visualise and to notice, record and verbalise detail but to give succinct and logical answers as well. They talked thoughtfully about interactions and relationships between shapes and patterns and ideas, but they also had an enthusiasm for accuracy and for verbatim information. All six students described themselves as visualisers and when asked what they remembered about a classroom in their first years at school, immediately described such things as the shape and proportions of the room, the placement of furniture and the colours of the pictures on the walls, often in great detail. Some had a bent towards model-making of one sort or another, for example the making of polyhedra, and made models which pleased the aesthetic and mathematical senses as well as the practical senses.

Students in Group S were among the most direct in expressing their opinions, and the most able not only to draw conclusions from the information supplied but also to want to discuss implications that followed from those conclusions. They clearly wanted to succeed at what they did but they did not seem to find individual competitiveness, for its own sake, a foolproof motivation for work. They had a total commitment to anything that fascinated them or caught their imagination. They were good at languages and computer programming, and were interested in philosophy. They expressed their ideas well, especially orally, and were acclaimed by peers as being good teachers.

Two students fitted into Group R. Both students philosophised less than those in Group S and had more difficulty visualising, or claimed not to do it at all. In mathematics, one liked to have opportunities to try new types of problems and to be able to obtain some originality in their solution. The other liked to have practical reasons for the work he did and the things he thought, and seemed to prefer being able to use the application of information he acquired for a purpose other than simply the pleasure of thinking, or of seeing the task completed. He made models of things that would work rather than of polyhedra. Both had skill in performance music. Neither had particularly good recall of an early classroom, but both were more able to tell something of what had had happened in school. Both students were described by their peers and some teachers as extremely able and fast in their calculations, less likely than others to worry about the elegance of proofs, and more likely to look for the answer than to be preoccupied with the method. They were described by teachers as not having been the most unusual of the mathematicians that that teacher had taught, in that they were more content with an algorithmic approach to mathematics than were those in Groups S or P., but they were recognised as having been the most able in their class, in terms of maths achievement.

The eight students in Groups S and R were all described as having been noticeably good at mathematics as early as their first two or three years at school, and some were described by their families as having exhibited quite complex or advanced mathematical skills as pre-schoolers. This was not so with the two students in Group P. Neither had been identified as able in mathematics at a particularly early age. Both could name an event that had triggered their interest in maths, and in both cases they had been around twelve years old at the time. They were the only two among those who described themselves as visualisers, who described an overview of their school rather than a description of the classroom. One described the relationship of blocks of classrooms to each other and to the playground. The other described the outside features of the building and added that it was "sort of in a valley between two hills". He said he used to learn by "making a clear picture of the shape of the object" but at university could not keep up with that so now learned by rote. They were able to rise to the occasion in competitive situations, and seemed more able to let individual success motivate them when necessary, than were those in Group S. Although they exhibited a strong visualisation component in the things they remembered, and in the way they approached mathematics, they did not appear to relate one detail to another in quite the way the others did.

Largely, the characteristics of the three groups corresponded with those identified by Krutetskii (1976). Group S tackled things much the same way as Krutetskii's Harmonic types had done, sometimes using an analytical approach, sometimes a geometric one, and almost always able to switch to the one which best suited the situation. Group R corresponded closely to Krutetskii's Analytical type. Although the term 'Geometric', given by Krutetskii to his third type, would not be the first term that one would apply to Group P, the characteristics described by Krutetskii in relation to mathematics, were very similar to those exhibited by these two students. The research reported by Osborn (1983), also identified varying profiles in mathematicians, reflective of the presence, in any individual, of different levels of attainment in four distinct, but not discreet, components of the thinking required for mathematical activity. As in this research, the transition from one type to another was seen as gradual rather than as absolute. Krutetskii recognised a similar trend, especially in regard to those of his sample whom he classified as being harmonic types.

Educational Implications. Krutetskii contended that there is such a thing as a "mathematical cast of mind" which can be seen clearly in pupils who are especially gifted in mathematics, and suggested at the end of his study that indications of this may be present at birth. One of the aspects of the present study which would support this, was the extent to which the pre-school inclinations of students from the three identified groups were reported differently from each other, but nonetheless consistently with that student's later development and mathematical skill.

The sort of indications given by Krutetskii's study, Osborn's research on profiles, and this research, have significant educational implications. They help to address questions of why one talented mathematics student will be bored in mathematics and feel that little of what is taught is stimulating, while another will be reasonably happy in the class even if they feel underextended. Providing appropriate approaches to learning mathematics, and providing extension work or acceleration, becomes a much more intricate task. Similarly, timetable planning will be effected by an understanding of which other curriculum subjects are likely to appeal to each group.

One syllabus in school mathematics, and one approach in teaching it, may not suit all, even from the earliest years at school. One student from Group R could add in 7s and 17s, memorise π to a number of decimal places, and work out the divisors of 24 before he went to school, but was unable to see the significance of cuisenaire rods in his first year at school and was only "middlingly" interested in making polyhedra when that was offered as extension work at Intermediate level. But a student from Group S who, on his own initiative, had sorted play blocks by colour, shape and size by fifteen months, and had taught himself to add, subtract, multiply and divide before he went to school, by observing the relationships of blocks, was quite at ease with cuisenaire rods, loved making polyhedra - and filled most of his Intermediate and Secondary school maths lessons by solving problems found from out-of-school sources.

A good deal of frustration and friction with teachers was reported by these students (and indeed by a number of others in the original survey). It may have developed because of the inability of some teachers to understand complex maths concepts themselves. But it may also have been exacerbated by the differences in approach between the student and the teacher, because of their respective differences of mathematical type. It became clear, for example, that what was 'proof' to one type, was not necessarily self-evident or easily accessible, to another. Osborn (Ibid, p37) notes that "the methodology of teaching adopted by a teacher, influenced by his or her own profile, is liable to favour an understanding of and communication to, pupils with similar profiles, [and] work to the disadvantage of pupils with strengths in other components." It was also the case that those in Group S learnt more easily, and expressed themselves with greater clarity, if there was a high oral component in their work. They were not always the quickest or the most motivated, especially at more advanced stages, to record their work in written form; but they could explain a greater variety of proofs, and switch more easily from one approach to another to suit the particular problem. This observation also has implications for examination methods, and for assumptions that the same test mark in relation to one student will indicate a similar level of ability in relation to another.

It is easy to assume that talented mathematicians will survive whatever they are taught, or will be able to acquire help at home if the education system cannot cater for them. Writers such as Howe (1990) indicate that, with a little training, parents of children of exceptional ability could do much more for them. One of the findings of this research, was that these young mathematicians most frequently came from families whose parents were skilled in different ways (note, for example, the number of Arts graduates among the parents) or were of a different type in their approach to problem-solving and discussion. The students in the study had largely outstripped their parents in mathematical knowledge and skill at a relatively early age. Often, they reported intellectual loneliness, even within their families. In these cases, they were dependent on books they could find themselves, on the stimulation and direction made available by the school, and on outside competitions, to retain the enthusiasm and competence their ability deserved. It should also be noted that, in group situations, they gained the most stimulation and made the greatest progress, when they worked with groups of their own type. Working with other types, when the differences were **not** noted and appreciated, not infrequently lead in itself to an undervaluing of a specific aptitude or skill.

A not infrequent reaction to research concerning types, has been that of assuming that mention of different types also implies that a value-judgement is being made. Overtones from the nurture/nature debate and from the vertical structuring of IQ tests, leads to the conclusion that the mention of types of minds and approach, will lead to one type of mathematician being valued over another. There is no need to assume this. Each approach has a vocational or intellectual value that means that the different skills and aptitudes do not need to be seen as being in competition.

References

Bloom, Benjamin S. (1985). Developing Talent in Young People. New York: Ballantine Books.

Borg, W.R., and Gall, M.D. (1983). Educational Research: An Introduction. (4th Edition) New York: Longman.

Curran, John, Holton, Derek, Marshall, Coralie, Pek, Wee Haur (1992). A Survey of Talented Secondary Mathematics Student. Dunedin: Dept. of Maths and Statistics, University of Otago.

Elley, W.B., and Irving, J.C. (1985). The Elley-Irving Socio-Economic Index 1981 Census Revision, New Zealand Journal of Educational Studies, 20 (2), 115-128.

Foster, W. (1986). The application of single subject research methods to the study of exceptional ability and extraordinary achievement. Gifted Child Quarterly, 30, (1), 33-37.

Gross, Miraca U.M. (1993). Exceptionally Gifted Children. London: Routledge.

Howe, Michael J.A. (1990) The Origins of Exceptional Abilities. Oxford: Basil Blackwell, Inc.

Kerry, Trevor (Editor) (1983). Finding and Helping the Able Child. London: Croom Helm.

Krutetskii, V.A. (1976). The Psychology of Mathematical Abilities in Schoolchildren. Chicago and London: University of Chicago Press.

Merriam, S.B. (1988). Case Study Research in Education: A Qualitative Approach. San Francisco: Jossey Bass.

Moore, John, and Morrison, Neil (1988). Someone Else's Problem? London: The Falmer Press.

Osborn, Herbert H. (1983). The Assessment of Mathematical Abilities, Educational Research, 25, (1), 28-40.

Renzulli, J.S. (1977). The Enrichment Triad Model: A Guide for Developing Defensible Programmes for the Gifted and Talented. Mansfield Centre, CT: Creative Learning Press.